A Determination of the Radiation Constant.

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According to the Stefan-Boltzmann law, the radiation emitted by a full radiator in surroundings at a temperature of absolute zero is proportional to the fourth power of the absolute temperature of the radiator, or $R = \sigma\theta^4$, where R = radiation in ergs per cm.^{2} per sec., θ = absolute temperature of radiator, σ = radiation constant. If the radiator is in surroundings at absolute temperature θ_1, which are themselves full radiators, then

$$R' = R_\theta - R_{\theta_1} = \sigma(\theta^4 - \theta_1{}^4),$$

where R' is the net radiation.

The first important determination of the radiation constant is due to Kurlbaum,* who obtained a value $5{\cdot}33 \times 10^{-5}$ erg/sec. cm.^{2} deg.^{4}, recently corrected to $5{\cdot}45 \times 10^{-5}$ erg/sec. cm.^{2} deg.^{4}.† Later investigations give results varying considerably from Kurlbaum's and from one another, and, on the whole, they indicate that Kurlbaum's value is too low. Some determinations are given in the following table:—

Observer.	Date.	$\sigma \times 10^5$.
Kurlbaum	1898	5·45
Féry	1909	6·30
Bauer and Moulin	1909	5·30
Valentiner	1910	5·47
Féry and Drecq	1911	6·51
Shakespear	1912	5·67
Gerlach	1912	5·80

The essential conditions in a determination of σ are, either that both emitter and receiver are full radiators, or that the amount by which they fall short of full radiators is known—an amount difficult to determine with certainty.

Hitherto, a measurement of σ in which both emitter and receiver were full radiators has not been made. Féry‡ and Féry and Drecq§, who have

* 'Ann. d. Phys.,' 1898, vol. 65.
† 'Ann. d. Phys.,' May, 1912.
‡ 'Compt. Rend.,' April 5, 1909.
§ 'Journ. d. Phys.,' July, 1911, p. 558.

obtained results $6 \cdot 3 \times 10^{-5}$ and $6 \cdot 5 \times 10^{-5}$ erg/sec. cm.^{2} deg.^{4}, used a blackened conical receiver, which is more nearly a full receiver than a plain black surface. Féry has pointed out that the advantage of this receiver depends upon the large number of reflections made by the incident radiation within the cone. At all points of incidence and reflection, diffusion, as well as reflection, takes place, which implies energy losses from the mouth of the cone. In this respect the instrument used by Féry is not a full receiver.

The nearest approach which we can make in practice to a full radiator or receiver consists in a good radiating surface in the form of a uniform temperature enclosure, with an aperture small compared with the total internal area.

The present paper describes some experiments in which the receiver fulfilled the above conditions. The emitter could only be considered an approximation to a full radiator. It consisted of a small Heræus furnace at a temperature of about 1000° C. The construction of such a furnace does not lend itself to the production of a perfectly uniform temperature enclosure. Steps taken to improve the furnace in this respect will be described later.

One of the chief objects of the paper is to describe the full receiver and to show that it is capable of giving consistent results. At present a furnace is being constructed which approximates more nearly to the ideal full radiator and it is intended to make further measurements with it when completed.

For the purpose of the experiment it is necessary to calculate the energy exchange between two fully radiating coaxial circular apertures at different temperatures, and at a distance apart of the same order as the diameters of the apertures themselves. This calculation is given in the appendix, where it is shown that

$$E = \tfrac{1}{4}\pi R' (r_1 - r_2)^2,$$

where E is rate of emission of energy from one of the apertures, R' is net radiation per cm.^{2}/sec, r_1 and r_2 the greatest and least distances between two points, one on each circle bounding the apertures.

In order to bring this expression into line with that used when the distance between the apertures is large compared with their diameters, it may be written

$$E = R' s_1 s_2 \{1 - (a^2 + b^2)/D^2\} / \pi D^2$$

as a first approximation, where s_1, s_2 are the areas of apertures; a, b, the radii of apertures; D, the distance between apertures.

If a and b are small compared with D, then the expression takes the familiar form in which it has been used by those investigators who have confined themselves to such distances that the correction $\{1 - (a^2 + b^2)/D^2\}$ was negligible.

In the arrangement to be described this expression was of the order 0·98, the radiation constant being calculated from the expression

$$E = \sigma\theta^4 s_1 s_2 \{1-(a^2+b^2)/D^2\}/\pi D^2,$$

where σ is the radiation constant and θ is the absolute temperature of furnace. The fourth power of the absolute temperature of the receiver was negligible in comparison with that of the furnace.

Description of Apparatus.

The receiver consisted essentially of an aniline thermometer with a bulb of about 2 litres capacity and a stem of 1 mm. bore. The bulb had double walls, between which was the aniline, the inner hollow cavity being lamp-blacked and provided with a circular orifice A which served as the receiving

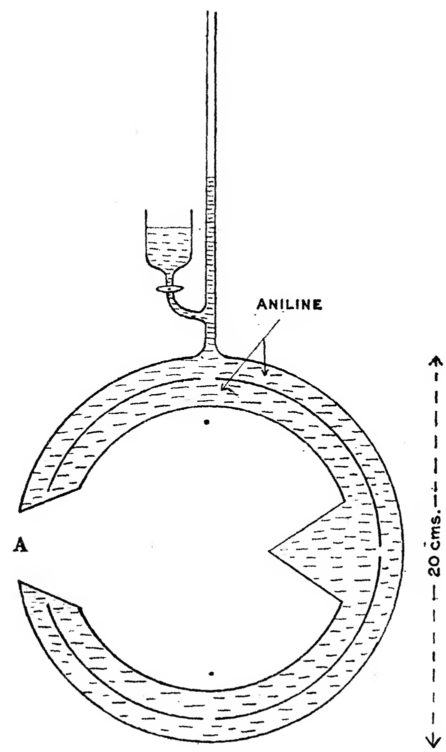

Fig. 1.

aperture (see fig. 1). The bulb was constructed of thin spherical copper shells, the inner one being 15 cm. in diameter, with an aperture of about 2 cm.². A third copper shell was placed in the liquid between the inner and outer shell to serve as a screen to prevent initial heat losses by direct radiation through the aniline from the inner surface. Radiation from the

furnace aperture was received by the aperture A (fig. 1), and fell on the inner blackened surface at the back of the bulb. In order to minimise energy losses by direct reflection, this region was occupied by a thin copper cone, so that no part of the receiving surface received rays normally. Since the thermometer contained a badly conducting liquid and also a screen to prevent radiation losses to the outside, the initial march of the meniscus in the stem gave a measure of the energy received per second; 1 mm. rise of the meniscus corresponded to a rise of temperature of 0·0005° C. In order to eliminate the effect of external temperature variations a companion thermometer was constructed, and both thermometer bulbs were enclosed in

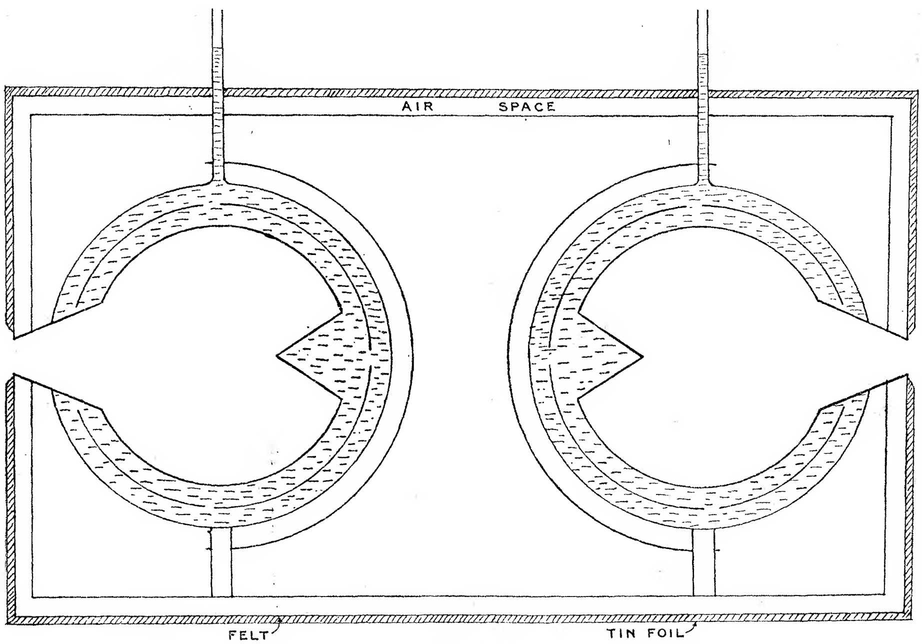

Fig. 2.

the same heat-insulating jacket, one alone receiving energy from the emitter. The jacket consisted of a double-walled zinc box well lagged on the outside with felt, the outer surface of which was covered with bright tinfoil (see fig. 2). In this way the rate of change of temperature of the two thermometers was made as small as possible when the room temperature was changing. The thermometers were now used differentially, allowance being made for the small difference in capacity and bore of capillaries. The march of the two meniscuses was observed prior to admitting radiation from the furnace into one of the apertures, and when the room temperature was changing slowly and uniformly, a screen was removed and the radiation

was admitted. The observations were now continued every half minute for 12 minutes, and the differential effect due to the energy from the furnace alone was determined.

The emitter consisted of a cylindrical electric furnace of the Heræus type, about 3·5 cm. in diameter and 20 cm. in length. Owing to the low conductivity of the porcelain tube on which was wound the platinum heating strip considerable temperature differences, visible to the eye at red heat, exist between each successive turn of the heating spiral. These differences were minimised by the introduction of a lining cylinder of nickel and two nickel plugs near the centre of the furnace (see fig. 3). In spite of this precaution a considerable fall of temperature takes place from the centre of the furnace to the aperture, and although the extremity of this region near

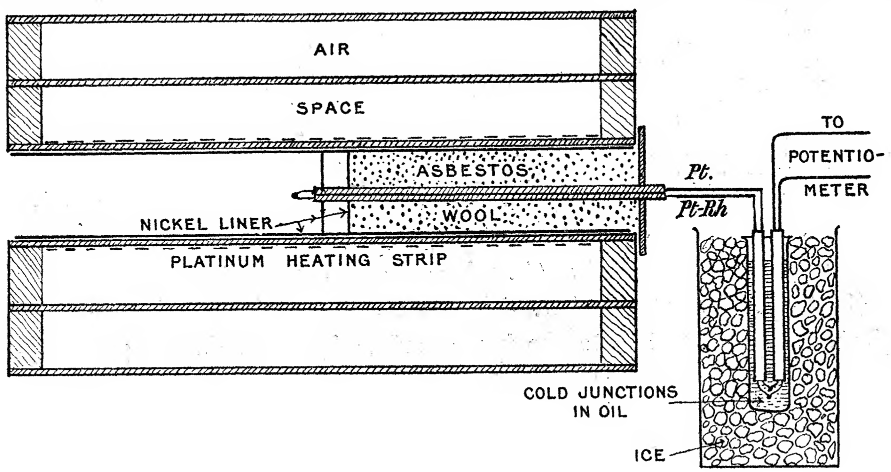

Fig. 3.

the aperture is not visible from the receiving aperture, such an enclosure does not constitute a full radiator. For this reason the results of the present measurements are offered as preliminary observations pending the construction of a new high temperature full radiator.

Screens.—Between the emitting and receiving apertures were two water-cooled screens coated with paraffin black on their surface (see fig. 4). In order to avoid heating of the furnace screen by the furnace a continuous stream of cold water from the town mains was brought in at the centre of this screen and conducted by means of a spiral guide to the extremities. To ensure an even flow of water through the screen placed before the receiver, the water was brought in at a number of peints at the bottom of the screen and taken out in a similar way at the top. The aperture in each screen consisted of a blackened cone of thin spun copper.

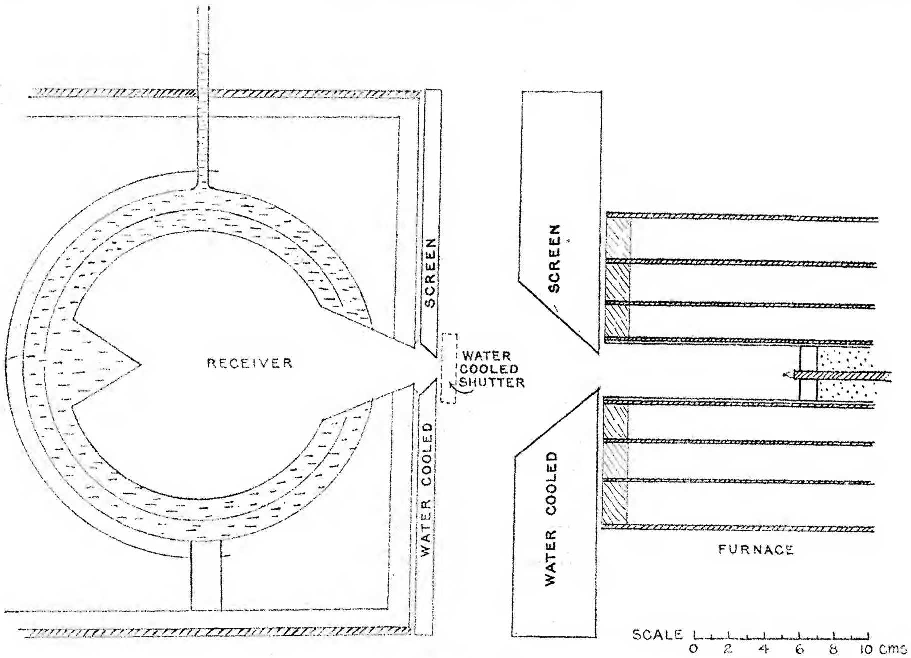

Fig. 4.

Method of Carrying out a Measurement.

(1) *Calibration of Receiver.*—The receiver contained a bare insulated platinum wire attached to the inner surface by a number of glass hooks.

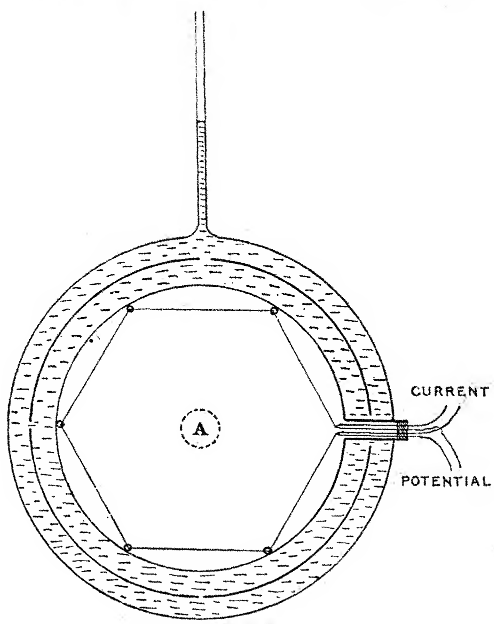

Fig. 5.

The wire formed a polygon lying close to the inner surface of the receiver; the plane of the polygon was symmetrically placed within the sphere and occupied a position parallel to the plane of the receiving aperture. The wire was provided with current and potential leads, and served as a means of introducing electrical energy for the purposes of calibration (see fig. 5).

The calibration was carried out as follows: When the apparatus indicated that the room temperature was changing slowly and uniformly the electric

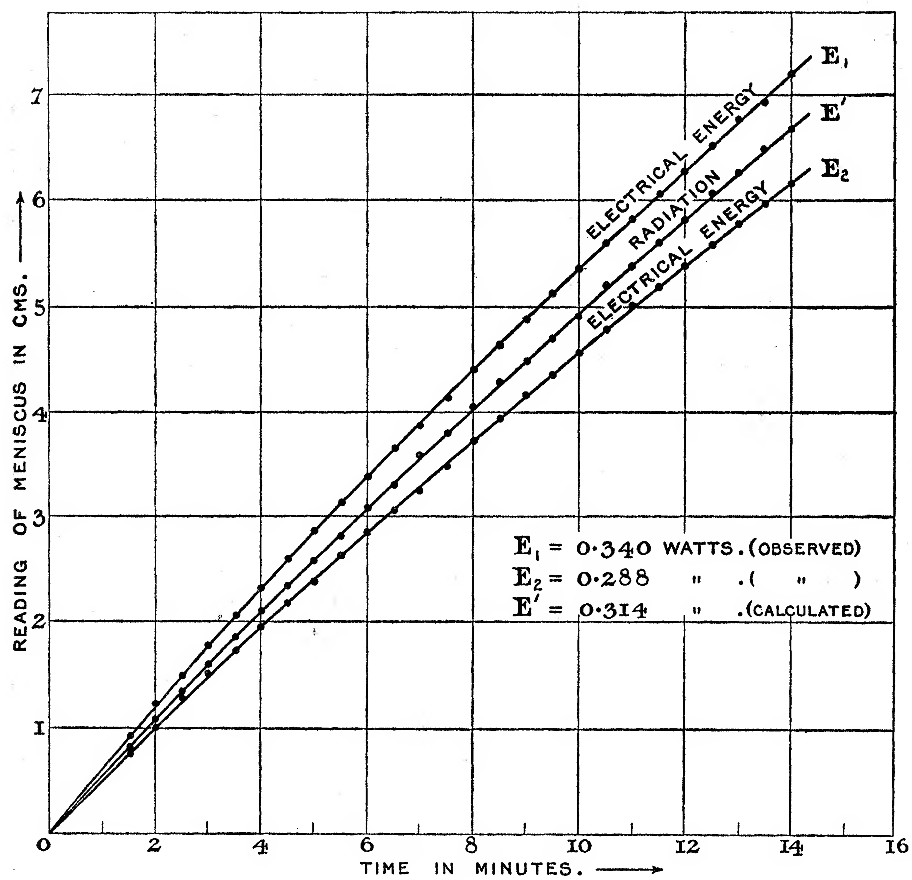

Fig. 6.

circuit was completed at a noted instant. The march of the meniscus both of the thermometer containing the electric circuit and of the companion thermometer was now observed every half-minute for 12 minutes. These readings were plotted against time, and the differential effect due to the energy received from the wire alone was determined. Two actual curves are shown on a reduced scale in fig. 6. By varying the electrical energy

a series of such curves was obtained (E_1, E_2, fig. 6). It was found that the ordinates of the successive curves at a given time t after the circuit was completed were proportional to the given energy supplies. This was true whatever the value of t, except at the beginning of the curves, where slight irregularities were apparent.

Twenty-four hours were allowed to elapse between each experiment. This was found to be necessary in order to allow both thermometers to come to the same temperature conditions.

(2) *Measurement of Energy Stream.*—Similar curves were obtained when energy was received by radiation from the furnace. Such a curve is shown at E′, fig. 6. The energy per second corresponding to the curve E′ could be readily obtained by interpolation from the neighbouring calibration curves. The initial portions of the curves were discarded and interpolations carried out at each of four successive different values of the abscissæ, viz., after 5, 8, 10, and 12 minutes, and the mean value taken.

Before the admission of radiation to the receiver from the furnace, the latter was allowed to run for four hours in order that it might reach a steady state. During this time a water-cooled zinc diaphragm was in position immediately in front of the receiving aperture. This screen, indicated by the dotted rectangle in fig. 4, consisted of a flat cylindrical box 4 cm. in diameter and 1 cm. thick, and carried a stream of running water at room temperature. Both sides of the screen were coated with paraffin black. When the furnace had attained a steady temperature, as indicated by the thermocouple within, the water-cooled diaphragm was swung out of position at a noted instant and the march of the meniscuses observed as already described.

(3) *Temperature of Furnace.*—This was measured by a platinum–platinum-rhodium thermocouple and potentiometer, the maximum error being from 1° to 2° in the neighbourhood of 1100° C. After use the couple was calibrated at the National Physical Laboratory, and the calibration was used in the present calculations.

In order to minimise electrical leakage from the furnace heating spiral to he thermocouple and potentiometer, the cylindrical nickel liner contained in the furnace and separating the thermocouple from the porcelain furnace tube was earthed. In spite of this precaution a small leakage was still evident. The effect of this leakage was eliminated by placing a reversing switch in the furnace circuit and determining the balance point on the potentiometer wire for both directions of the furnace current. The difference in the balance point never exceeded 1° C.

Calculation of Results.

A series of ten curves was obtained for the purposes of calibration, the electrical energy supply varying from 0·28 to 0·42 watts. A series of five curves was obtained by admitting radiation from the furnace, the calculated energy supply varying from 0·28 to 0·34 watts.

The areas of the radiating and receiving apertures remained constant, while their distance apart and the temperature of the furnace varied from one experiment to another.

The results are embodied in the following table :—

Table I.

Area of furnace aperture $= 2{\cdot}800$ cm.^{2}.
Area of receiving aperture $= 1{\cdot}070$ cm.^{2}.

Temperature of furnace.	Distance between apertures.	Radiation received calculated from successive ordinates.					$\sigma \times 10^5$.
		5 mins.	8 mins.	10 mins.	12 mins.	Mean.	
°C.	cms.	watts.	watts.	watts.	watts.	watts.	erg/sec. cm.^{2} deg.^{4}.
1117	8·289	0·300	0·300	0·300	0·305	0·301	5·93
1115	8·289	0·296	0·297	0·297	0·297	0·297	5·86
1119	8·266	0·303	0·303	0·304	0·304	0·304	5·90
1120	8·266	0·305	0·305	0·305	0·305	0·305	5·90
1097	8·266	0·283	0·283	0·283	0·283	0·283	5·87

Mean value, $\sigma = 5{\cdot}8\,(9) \times 10^{-5}$ erg/sec. cm.^{2} deg.^{4}.

Discussion of Results.

An objection to the present method of experiment lies in the fact that the energy to be measured and the energy used for the purposes of calibration are not introduced into the receiver in the same manner. An attempt to show that the objection is not serious was made by varying the position of the heating coil. If the cavity were a perfectly "black body," the resulting effect should be independent of this position. This point was tested in the following way: A separate heater, consisting of a single platinum wire, provided with the usual current and potential leads, was stretched across a diameter within the receiver. This position is very different from that of the original heating coil already described. A series of calibration curves was obtained, using this wire to carry the current, and the energy of the radiation streams from the furnace was calculated from these observations. The results differed by less than 1 per cent. from those obtained with the original heating coil. The small differences indicate that the energy losses

through the aperture are slightly greater when the diametric heating wire is used. Such an effect is to be expected, since this wire is stretched across the middle of the receiver, a position more favourable for direct radiation losses through the aperture. Owing to this fact, the calibration data obtained with the diametric heater were discarded.

It will be seen that the method of measuring the energy received is essentially the same as that used by Féry and Drecq,* but differs in two important respects. Firstly, the energy falls on a full receiver, and, secondly, the difficulty of temperature variation is avoided by using two such receivers differentially. Again, in Féry and Drecq's conical receiver the heating coil used for calibration was wound on the outer surface of the cone and was in contact with the liquid in the thermometer bulb. With a conical receiver the proportion of the calibrating energy retained will depend entirely upon the position of the heating coil, while in the present method there is evidence to show that this proportion is practically independent of the position of the coil. This fact makes it highly probable that equal quantities of energy put in as electrical energy and as radiation would produce the same effect.

Sources of Error.

In all previous determinations of the value of the radiation constant the receiver has not been a "black body." In some cases the arrangement of the screens has been such as to allow of additional energy reaching the receiver by radiation from the screens. The first point has already been discussed. With regard to the second it has been pointed out by a previous writer† that one of the most important screens in the arrangement of Féry and Drecq was not water cooled, and that it consisted of thick blackened cork containing a cylindrical hole, and from the surface of the cork additional energy would reach the receiving aperture by reflection. The additional energy received would make the resulting value of σ too high. In the arrangement described in the present paper (see fig. 4) additional radiation would reach the receiving aperture after undergoing two successive reflections. The screen facing the furnace would reflect some of the radiation it received from the furnace aperture to the screen facing the receiving aperture. This, again, would reflect to the receiving aperture. If the screens were not blacked, but left with a tarnished metal surface, reflecting, say, 50 per cent. of the incident energy, the writer calculated that σ would be 20 per cent. too high. This point was verified experimentally, the results of three such experiments being $\sigma = 7{\cdot}1, 7{\cdot}0$, and $7{\cdot}0 \times 10^{-5}$ erg/sec. cm.^{2} deg.^{4}.

* 'Journ. d. Phys.,' July, 1911, p. 558.

† Shakespear, 'Roy. Soc. Proc.,' 1912, A, vol. 86, p. 180.

In all the experiments in Table I the screens were blacked. If it is assumed that the blackened surface used diffuses 5 per cent. of the energy it receives, then it may be shown by making similar approximations to those in the previous calculation that the value of σ would not be increased by more than 1 per cent.

Another source of error is due to the water-cooled cone of the furnace screen reflecting some radiation from the cooler part of the furnace into the receiving aperture. The calculation of the additional energy received by this means presents some difficulty, but a rough approximation shows that it is not more than 2 per cent. of the whole. The difficulty due to reflection from this cone may be overcome by reversing the furnace screen. This was impracticable in the present case, since the placing of the furnace farther back would diminish the pencil of radiation which must come from that region of the furnace in the neighbourhood of the thermocouple; and the apparatus was not capable of dealing with smaller quantities of energy with a very high degree of accuracy. For this and other reasons the screen was not reversed, but when the new high temperature full radiator is constructed, arrangements will be made to overcome the difficulty.

It may be pointed out that the error due to reflection from the cone will be compensated at least partially by that due to the furnace not being a full radiator.

The experiment has grown out of a suggestion made to me by Prof. J. H. Poynting, F.R.S., that in the measurement of σ it was desirable that both radiator and receiver should be full radiators; and to him and to Dr. Guy Barlow I wish to express my gratitude for their kindly interest throughout the work.

Appendix.

Calculation of the Energy Exchange between Two Fully Radiating Coaxial Circular Apertures at Different Temperatures.

For the following I am largely indebted to Mr. C. J. Lay.

The mutual illumination* of two such apertures of intrinsic brightness I may be written

$$\tfrac{1}{2}\mathrm{I}\iint \log r \cos\epsilon \, ds\, ds',$$

where ϵ is the angle between the tangents at ds, ds', and r is the distance between the elements ds, ds', of the boundaries. In this case put

$$ds = a d\theta \quad \text{and} \quad ds' = a\, d\theta',$$

then $$\cos\epsilon = \pm\cos(\theta-\theta'), \qquad r^2 = z^2+a^2+b^2-2ab\cos\epsilon.$$

* *Cf.* Hermann's 'Geometrical Optics,' p. 212.

For integration round the first circle put $\theta' = 0$. Then $\theta = \epsilon$ and we have

$$\tfrac{1}{4}\mathrm{I}a \iint \log(z^2+a^2+b^2-2ab\cos\theta)\cos\theta\, ds'\, d\theta.$$

Now
$$\int_0^{2\pi} \log(\alpha-\beta\cos\theta)\cos\theta\, d\theta$$
$$= \Big[\sin\theta\log(\alpha-\beta\cos\theta)\Big]_0^{2\pi} - \beta\int_0^{2\pi}\{\sin^2\theta/(\alpha-\beta\cos\theta)\}\, d\theta$$
$$= -\beta\int_0^{2\pi}\{(\cos^2\theta-1)/(\beta\cos\theta-\alpha)\}\, d\theta$$
$$= -\beta\int_0^{2\pi}\left\{\frac{\cos\theta}{\beta}+\frac{\alpha}{\beta^2}+\left(\frac{\alpha}{\beta^2}-1\right)\Big/(\beta\cos\theta-\alpha)\right\} d\theta$$
$$= -2\frac{\pi\alpha}{\beta}+2\frac{\alpha^2-\beta^2}{\beta}\int_0^{\pi}\frac{d\theta}{\alpha-\beta\cos\theta}.$$

Now
$$\int_0^{\pi}\frac{d\theta}{\alpha-\beta\cos\theta} = \frac{\pi}{\sqrt{(\alpha^2-\beta^2)}}.$$

Therefore we have

$$\tfrac{1}{4}\mathrm{I}a\left\{-2\pi\frac{\alpha}{\beta}+2\frac{\pi}{\beta}\sqrt{(\alpha^2-\beta^2)}\right\}$$

after the first integration, where

$$\alpha = z^2+a^2+b^2 \quad\text{and}\quad \beta = 2ab.$$

For the second integration the result must be the same whatever value of θ' we start with, hence we have only to replace ds' by $2\pi b$. We then get

$$\pi^2\frac{ab}{\beta}\{\sqrt{(\alpha^2-\beta^2)}-\alpha\}.$$

This gives the sign negative, which shows we ought to change the sign of $\cos\epsilon$; we then get

$$\tfrac{1}{2}\pi^2\mathrm{I}\{\alpha-\sqrt{(\alpha^2-\beta^2)}\} = \tfrac{1}{4}\pi^2\mathrm{I}\{2(z^2+a^2+b^2)-2\sqrt{[(z^2+a^2+b^2)^2-4a^2b^2]}\}$$
$$= \tfrac{1}{4}\pi^2\mathrm{I}(r_1-r_2)^2,$$

where
$$r_1^2 = z^2+(a+b)^2, \qquad r_2^2 = z^2+(a-b)^2.$$

Since $\mathrm{I} = \mathrm{R}/\pi$, we have

$$\mathrm{E} = \tfrac{1}{4}\pi\mathrm{R}/(r_1-r_2)^2.$$